

Leptonic widths of high excitations in heavy quarkonia

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Abstract

Agreement with the measured electronic widths of the $\psi(4040)$, $\psi(4415)$, and $\Upsilon(11019)$ resonances is shown to be reached if two effects are taken into account: a flattening of the confining potential at large distances and a total screening of the gluon-exchange interaction at $r \gtrsim 1.2$ fm. The leptonic widths of the unobserved $\Upsilon(7S)$ and $\psi(5S)$ resonances: $\Gamma_{e^+e^-}(\Upsilon(7S)) = 0.11$ keV and $\Gamma(\psi(5S)) \approx 0.54$ keV are predicted.

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I. INTRODUCTION

Recently, new experimental data on leptonic widths in heavy quarkonia (HQ) has been presented [1, 2, 3]. In the BaBar experiment the mass and the total and electronic widths of the $\Upsilon(10580)$ resonance have been measured with great accuracy [1], while the CLEO Collaboration has observed significantly larger muonic branching ratios of the $\Upsilon(nS)$ resonances ($n = 1, 2, 3$) compared to the values adopted till now [4]. Besides, existing experimental data on the total cross section for hadron production in e^+e^- annihilation (in the region $\sqrt{s} = 3.8 \div 4.8$ GeV) have been reanalysed [3], and the total and electronic widths of the $\psi(4040)$, $\psi(4160)$, $\psi(4415)$ resonances are shown to be larger by 20% for the $\psi(4040)$ and by 70% for the $\psi(4415)$ resonance than their values from the Particle Data Group (PDG) [4].

Accurate knowledge of the leptonic widths of high meson excitations is of special importance for the theory, because the wave functions (w.f.) at the origin of vector nS resonances, $|R_n(0)|^2$, proportional to $\Gamma_{e^+e^-}$, directly provides information about the static $Q\bar{Q}$ interaction at all distances including large r . In HQ they can be expressed through the matrix element (m.e) of the static force. Unfortunately, the true behavior of the static potential $V(r)$ at $r \gtrsim 1$ fm is still undefined in QCD and from lattice measurements there is only an indication that the confining linear potential $\sigma_0 r$ is becoming more flat at large r [5].

The origin of this phenomenon has been discussed in [6] where it was shown that flattening occurs due to the creation of virtual $q\bar{q}$ pairs in the Wilson loop before the string breaking takes place [6]. Due to the presence of virtual loop(s) the surface of $\langle W(C) \rangle$, and therefore the effective string tension, is becoming smaller: the string tension $\sigma(r)$ depends on r and its derivative $\sigma'(r) < 0$. For light mesons this phenomenon gives rise to a correlated shift down of radial excitations which increases with n (for $\rho(3S)$ this shift is about 150 MeV [6]). Our chereent alculations show that a similar *correlated* shift down takes place for high excitations in HQ, being about 60 MeV for $\psi(4415)$ and about 30 MeV for $\Upsilon(6S)$ [7], while the leptonic widths of high excitations provide an additional opportunity to test the confining potential at large r .

In this paper we concentrate on the leptonic widths of HQ. For low-lying resonances they have been calculated in many papers [8-10], where it has been observed that agreement with the experimental values of $\Gamma_{e^+e^-}$ can be obtained only if the asymptotic freedom (AF) behaviour of the vector coupling $\alpha_V(r)$ in the gluon-exchange (GE) interaction is taken into

account (this effect is about 50%). For the Coulomb interaction ($\alpha_V = \text{const}$) the leptonic widths of both low- and high-lying resonances in the Υ - and ψ -families appear to be 50-100% higher than their experimental values.

However, even if the AF behavior of $\alpha_V(r)$ is taken into account and for low-lying resonances (like J/ψ , $\psi(2S)$, $\Upsilon(nS)$ ($n \leq 3$)) the leptonic widths are in agreement with experiment, still for very high excitations (like the $\psi(4040)$, $\psi(4415)$, $\Upsilon(11019)$) the calculated $\Gamma_{e^+e^-}$ appear to be significantly larger than the experimental values. The characteristic feature of these resonances is that they have very large sizes (their r.m.s. radii $r_n \gtrsim 1.2$ fm) and therefore their w.f. at the origin are very sensitive to details of the $Q\bar{Q}$ interaction at all distances. It does not seem accidental that better agreement with experiment for high resonances is obtained in [11] where a mild (logarithmic) confining potential (instead of linear $\sigma_0 r$ potential) has been used.

In this paper we study two effects which give rise to a decrease of the leptonic widths of high excitations in HQ. The first one is the flattening of the confining potential at large r . The second effect occurs if the GE interaction is very much suppressed or even switched off due to a screening at distances $r \gtrsim 1.0$ fm. The reason of such a total screening needs a special analysis [7], but the dynamics of a resonance with large radius, defined by the confining potential only, appears to be rather simple.

II. LEPTONIC WIDTHS AS PROBES OF THE GLUON EXCHANGE INTERACTION

The electronic width of the vector meson $V(nS)$ is given by the Van Royen–Weisskopf formula [12] with the QCD correction taken into account [13]. It contains the w.f. at the origin and some known quantities:

$$\Gamma_{e^+e^-}(V(nS)) = \frac{4e_Q^2\alpha^2}{M_n^2(V)} |R_n(0)|^2 \left(1 - \frac{16}{3\pi}\alpha_s(2m_Q)\right), \quad (2.1)$$

Here for $\alpha_s(2m_c)$ and $\alpha_s(2m_b)$ we use the conventional values: $\alpha_s(2m_c) = 0.253$, $\alpha_s(2m_b) = 0.177$ (e.g. see [10]). The w.f. at the origin is proportional to the leptonic width and on the other hand it can be expressed through the m.e. over the static force $F(r) = \frac{dV}{dr}$.

In the nonrelativistic (NR) approximation the relation is [14]

$$|R_n^{NR}(0)|^2 = m_Q \langle F(r) \rangle_{nS}. \quad (2.2)$$

Here for m_Q the heavy quark pole mass entering NR Hamiltonian must be used [15]. For relativistic kinematics and a relativistic Hamiltonian with the use of the “einbein approximation” for the spinless Salpeter equation instead of Eq. (2.2) the following relation can be obtained [16]:

$$|R_n(0)|^2 = \omega_Q \langle F(r) \rangle_{nS}, \quad (2.3)$$

where

$$\omega_Q(nS) = \langle \sqrt{\mathbf{p}^2 + m_Q^2} \rangle_{nS} \quad (2.4)$$

is the average kinetic energy of a heavy quark, or the quark constituent mass. For c and b quarks in HQ the difference between ω_Q and the pole mass m_Q is about 200 MeV for low-lying states and about $250 \div 300$ MeV for high excitations and this difference gives about 20% (5%) corrections to $|R_n(0)|^2$ in charmonium (bottomonium).

In the general case the static potential can be presented in the form

$$V(r) = r\sigma(r) - \frac{4}{3} \frac{\alpha_V(r)}{r} f_{\text{scr}}(r). \quad (2.5)$$

To describe low-lying states (below the open-flavor threshold) it is sufficient to take a linear confining potential with $\sigma(r) = \text{const} = \sigma_0$ and to put the screening function $f_{\text{scr}}(r) = 1$. For high-lying resonances both effects—the flattening of the confining potential and the screening of GE interaction—are becoming important. We shall consider the effects coming from screening in detail in our next paper [7], while here we take the screening function.

$$f_{\text{scr}} = \begin{cases} 1, & r < R_{\text{scr}}, \\ f_0 \exp(-(\sqrt{\sigma} r)^{4/3}), & r \geq R_{\text{scr}}. \end{cases} \quad (2.6)$$

The choice of this function with $R_{\text{scr}} \approx 0.6$ fm is motivated by the analysis of the screening effects in [17]. Here we take a larger value for the screenig radius: $R_{\text{scr}} \approx 1.0$ fm.

Then for low and high excitations one can use different approximations in Eq. (2.3). For low excitations $\sigma'(r)$ is negligible, $\langle \sigma(r) \rangle \approx \sigma_0$, but the contribution from the derivative $\alpha'_V(r)$ in Eq. (2.3) is important (it reflects the influence of the AF behavior of the coupling $\alpha_V(r)$) and one obtains

$$|R_n(0)|^2 = \omega_Q(n)\sigma_0 + \frac{4}{3}\omega_Q(n) \left\{ \langle r^{-2}\alpha_V(r) \rangle_{nS} - \langle r^{-1}\alpha'_V(r) \rangle_{nS} \right\} \quad (\text{small } n). \quad (2.7)$$

For high-lying excitations, on the contrary, the derivative $\alpha'_V(r)$ is small and, moreover, in bottomonium the negative term $\langle r\sigma'(r) \rangle_n$ also remains much smaller than $\langle \sigma(r) \rangle$. For such

resonances effects from the screening of GE interaction is becoming important:

$$|R_n(0)|^2 = \omega_Q(n) \{ \langle \sigma(r) \rangle_{nS} - \langle r\sigma'(r) \rangle_{nS} + \frac{4}{3} \langle r^{-2} \alpha_V(r) f_{\text{scr}}(r) \rangle_{nS} - \frac{4}{3} \langle r^{-1} \alpha_V(r) f'_{\text{scr}}(r) \rangle_{nS} \}. \quad (2.8)$$

In Tables I and II we present the HQ leptonic widths calculated for three potentials still neglecting the screening effects:

1. For the Cornell potential with the parameters taken from [8] ($\alpha_V(r) = \text{const} = 0.52$) the leptonic widths are very large, being 50÷ 70% larger for all $\Upsilon(nS)$ resonances ($n \leq 6$) than the experimental values.

2. For the potential taken from [15],

$$V_B(r) = \sigma_0 r - \frac{4}{3} \frac{\alpha_B(r)}{r}, \quad (2.9)$$

the vector coupling $\alpha_B(r)$ is defined in background perturbation theory. This vector coupling $\alpha_B(r)$ has the correct perturbative limit at small distances and also possesses the property of freezing at large r . As seen from Tables 1 and 2 this potential gives a good description of the electronic widths for many HQ states: $J/\psi, \psi(3686)$ in charmonium and for all $\Upsilon(nS)$ resonances with exception of the $\Upsilon(6S)$ resonance (the mass $M_{\text{exp}}(6S) = 11019$ MeV).

3. To demonstrate the sensitivity of the leptonic widths to the behavior of the confining potential at large distances we consider the “modified” potential,

$$V_M(r) = r\sigma(r) - \frac{4}{3} \frac{\alpha_B(r)}{r}, \quad (2.10)$$

where the flattening effect is taken into account and the string tension $\sigma(r)$ is taken as for light mesons [6] while the vector coupling $\alpha_B(r)$ is the same as in the potential Eq. (2.9).

From Tables I and II one can see that for the modified potential Eq. (2.10) the leptonic widths of the $J/\psi, \psi(2S)$ and $\Upsilon(nS) (n \leq 5)$ are practically the same as for the linear potential $\sigma_0 r$ Eq. (2.9) while for the higher resonances ($\psi(4040), \psi(4415)$, and $\Upsilon(11019)$) they are smaller by only $\sim 10\%$ and still exceed $\Gamma_{e^+e^-}(\text{exp})$. The characteristic feature of these three resonances is their large sizes (even in single-channel approximation) $r_3(\psi(3S)) \approx 1.2$ fm; $r_4(\psi(4S)) \approx 1.4$ fm, $r_6(\Upsilon(6S)) \approx 1.4$ fm. There can be two possible reasons for a further decrease of their leptonic widths. First, one may think of the coupling of the considered $Q\bar{Q}$ resonance to an open meson-meson channel. Comparison of the experimental data with our calculations show that the $4S$ state, $\Upsilon(10580)$, has a hadronic shift of about

TABLE I: The leptonic widths (in keV) of the $\Upsilon(nS)$ resonances for the Cornell potential and the potentials given by Eqs. (2.9) and (2.10).

Potential	1S	2S	3S	4S	5S	6S
$\sigma_0 r - \frac{\kappa}{r}$ ^{a)}	2.60	0.94	0.66	0.54	0.47	0.42
$\sigma_0 r - \frac{4}{3} \frac{\alpha_B(r)}{r}$ ^{b)}	1.21	0.56	0.41	0.34	0.30	0.27
$\sigma(r)r - \frac{4}{3} \frac{\alpha_B(r)}{r}$ ^{c)}	1.14	0.54	0.40	0.32	0.27	0.24
experiment [4]	1.32(7)	0.52(8)	0.48(11)	0.248(31)	0.31(7)	0.130(30)
[2]				0.321(46)		

^{a)} From [8] where $\sigma_0 = 0.1826 \text{ GeV}^2$, $\kappa = 0.52$; $m_b = 5.17 \text{ GeV}$ is in fact the constituent mass ω_b

Eq. (2.4).

^{b)} Here $\sigma_0 = 0.18 \text{ GeV}^2$, $\alpha_B(r)$ is taken from [15], where $\Lambda_{\overline{MS}}(2 - \text{loop}) = 242 \text{ MeV}$ ($n_f = 5$), and the pole mass $m_b(2 - \text{loop}) = 4.83 \text{ GeV}$.

^{c)} Here $\sigma(r) = \sigma_0 g(r)$ is taken from [6] with $\sigma_0 = 0.18 \text{ GeV}^2$, ($g(0) = 1$), $\alpha_B(r)$ is taken as in footnote ^{b)}.

TABLE II: The leptonic widths (in keV) of the $\psi(nS)$ resonances for the same potentials as in Table I.

Potential	1S	2S	3S	4S
$\sigma_0 r - \frac{\kappa}{r}$ ^{a)}	8.18	3.68	2.62	2.01
$\sigma_0 r - \frac{4}{3} \frac{\alpha_B(r)}{r}$ ^{b)}	5.13	2.48	1.80	1.39
$\sigma(r)r - \frac{4}{3} \frac{\alpha_B(r)}{r}$ ^{c)}	5.10	2.42	1.70	1.18
experiment [4]	5.26(37)	2.19(15)	0.75(15)	0.47(10)
[3]			0.89(8)	0.71(10)

^{a)} The parameters of the Cornell potential are the same as in footnote ^{a)} in Table I and

$m_c = 1.84 \text{ GeV}$.

^{b)} See footnote ^{b)} in Table I; the pole mass $m_c = 1.44 \text{ GeV}$, $\Lambda_{\overline{MS}} = 260 \text{ MeV}$ ($n_f = 4$).

^{c)} See footnote ^{c)} in Table I; the pole mass $m_c = 1.44 \text{ GeV}$, $\Lambda_{\overline{MS}} = 260 \text{ MeV}$ ($n_f = 4$).

50 MeV due to coupling to the $B\bar{B}$ channel [7], nevertheless, the calculated electronic width (see Table I) appears to be in very good agreement with the new precision measurements of $\Gamma_{e^+e^-}(\Upsilon(10580))$ [1]. Also for the $\Upsilon(10865)$, the $5S \ b\bar{b}$ state, for which the mass is close to the $B_s^* \bar{B}_s^*$ threshold, agreement between the calculated and experimental value of $\Gamma_{e^+e^-}$

is obtained. So, one can assume that open channels do not drastically change the leptonic width of a resonance considered and cannot explain the $\sim 70\%$ difference between the theoretical and experimental leptonic widths for $\psi(4415)$ and $\Upsilon(11019)$. (A small mixing of the $Q\bar{Q}$ and meson-meson channels was also observed in Lattice QCD (second ref. [5])).

Therefore we assume here that a significant reduction of the leptonic widths of the $\psi(4415)$ and $\Upsilon(11019)$ resonances occurs due to a change in the static potential: a screening of the GE interaction at large r and flattening of the confining potential.

III. LEPTONIC WIDTHS OF HIGHLY EXCITED RESONANCES

If the screening of the GE interaction with $f_{\text{scr}}(r)$ Eq. (2.7) is taken into account, then the w.f. at the origin is defined by the relation Eq. (2.9) where the contribution from the GE term appears to be small ($< 10\%$) for high excitations, so that

$$|R_n(0)|^2 = \omega_Q(n) \{ \langle \sigma(r) \rangle_n - \langle r\sigma'(r) \rangle_n \} \equiv \omega_Q(n) \bar{\sigma}_n \quad (3.1)$$

Here we shall use the function $\sigma(r)$ in the form and with the parameters suggested in [6]. Its characteristic values are following:

$$\begin{aligned} \sigma(r) &\approx \sigma_0 \quad \text{for } r \lesssim 1 \text{ fm}, \\ \sigma(r = 1.3 \text{ fm}) &\approx 0.94 \sigma_0, \\ \sigma(r = 2.5 \text{ fm}) &\approx 0.78 \sigma_0, \\ \sigma(r \gtrsim 4 \text{ fm}) &= 0.6 \sigma_0. \end{aligned} \quad (3.2)$$

i.e. this string tension is slowly decreasing for larger $Q\bar{Q}$ separations r and has asymptotic value $\sigma_{\text{asym}} = 0.6\sigma_0 (\approx 0.11 \text{ GeV}^2 \text{ for } \sigma_0 = 0.18 \text{ GeV}^2)$. Our flattening confining potential continues to grow (with a smaller slope), and it significantly differs from the one suggested in [18], where the confining potential is taken as a constant equal to $R_{SC} \sigma(R_{SC})$ for $r \geq R_{SC} \approx 1.6 \text{ fm}$. One may notice that with such an assumption the quarks in a meson are not confined and can be liberated.

For this simple asymptotic potential $V_{\text{asym}}(\text{large } r) = r\sigma(r)$ the constituent masses $\omega_n(Q\bar{Q})$ and $\langle \sigma(r) \rangle_{nS}$ can be calculated easily from the solutions of the spinless Salpeter equation [15]. For the $\Upsilon(6S)$ and $\Upsilon(7S)$, using ($\sigma_0 = 0.18 \text{ GeV}^2$ and $m_b \approx 4.83 \text{ GeV}$) the

following numbers are obtained,

$$\begin{aligned}\omega_7(b\bar{b}) &\approx \omega_6(b\bar{b}) = 5.1 \text{ GeV}, \\ \langle \sigma(b\bar{b}, r) \rangle_{6S} &= 0.171 \text{ GeV}^2 \\ \langle \sigma(b\bar{b}, r) \rangle_{7S} &= 0.167 \text{ GeV}^2.\end{aligned}\tag{3.3}$$

and the term $\langle r\sigma'(r) \rangle$ is relatively small. Then from Eq. (3.1)

$$\begin{aligned}|R_6(b\bar{b}, 0)|^2 &= 0.87 \text{ GeV}^3, \\ |R_7(b\bar{b}, 0)|^2 &= 0.85 \text{ GeV}^3.\end{aligned}\tag{3.4}$$

and from Eq. (2.1) one obtains

$$\Gamma_{e^+e^-}(\Upsilon(11.019)) = 0.12 \text{ keV}.\tag{3.5}$$

This value is in good agreement with the experimental value $\Gamma_{e^+e^-}(\Upsilon(6S)) = 0.130 \pm 0.030$ keV [4]. With the use of Eq. (3.8) the electronic width of the still unobserved $\Upsilon(7S)$ can also be predicted:

$$\Gamma_{e^+e^-}(\Upsilon(7S)) = 0.11 \text{ keV},\tag{3.6}$$

where the value of the mass, $M_7 = 11.25$ GeV (obtained in single-channel approximation) has been used. Note that the mass difference $M(7S) - M(6S)$ is not small, about 230 MeV.

In charmonium for better accuracy, the negative correction in $\bar{\sigma}_n$ Eq. (2.10) coming from the derivative $\langle r\sigma'(r) \rangle$, is becoming larger and gives a contribution of $\sim 15\%$. The value of $\bar{\sigma}$ for the $\psi(4S)$ is $\langle \sigma(r) - r\sigma'(r) \rangle_{4S} = 0.14 \text{ GeV}^2$ and $\langle \sigma(r) - r\sigma'(r) \rangle_{5S} = 0.13 \text{ GeV}^2$ for $\psi(5S)$, while the constituent masses are: $\omega_4(c\bar{c}) = 1.71 \text{ GeV}$ and $\omega_5(c\bar{c}) = 1.67 \text{ GeV}$. Then from Eq. (3.1) it follows that

$$|R_4(c\bar{c}, 0)|^2 = 0.24 \text{ GeV}^3, |R_5(c\bar{c}, 0)|^2 = 0.22 \text{ GeV}^3,\tag{3.7}$$

and correspondingly, the electronic widths are

$$\Gamma_{e^+e^-}(\psi(4415)) = 0.66 \text{ keV}, \quad \Gamma_{e^+e^-}(\psi(5S)) = 0.54 \text{ keV}.\tag{3.8}$$

For the $\psi(4415)$ resonance our theoretical prediction in Eq. (3.8) agrees very well with that from the analysis of Seth [3] ($\Gamma_{e^+e^-}(\psi(4415))_{\text{exp}} = 0.71 \pm 0.10 \text{ keV}$), while both numbers significantly differ from PDG's $\Gamma_{e^+e^-}(\psi(4415)) = 0.47 \pm 0.10 \text{ keV}$.

TABLE III: The leptonic widths (in keV) of highly excited states in charmonium and bottomonium for the flattening potential $\sigma(r)r$, taken from [6] ($\sigma_0 = 0.18 \text{ GeV}^2$).

	$\psi(4040)$	$\psi(4415)$	$\psi(5S)$ $M \approx 4630 \text{ MeV}$	$\Upsilon(11019)$	$\Upsilon(7S)$ $M \approx 11250 \text{ MeV}$
this paper	0.94	0.66	0.54	0.12	0.11
Exper. [4]	0.75(15)	0.47(10)		0.13(3)	
[3]	0.89(8)	0.71(10)			

Our treatment above was done in single-channel approximation when the possibility of string breaking is neglected, while the creation of virtual $q\bar{q}$ pairs is taken into account through the dependence of $\sigma(r)$ on r . Since at present there is no fundamental string-breaking theory in QCD, we neither do know what the probability of string breaking and is nor do we know the probability of the existence of very high $Q\bar{Q}$ excitations. Therefore we do not know what is the upper limit, or the admissible size R_{max} of a high-lying resonance (the $Q\bar{Q}$ string) above which a resonance cannot exist.

Still, the resonances $\Upsilon(7S)$ and $\psi(5S)$ do not have large sizes, the splitting $\Upsilon(7S) - \Upsilon(6S)$ is not small, ($\Delta M \sim 230 \text{ MeV}$), and therefore one may expect them to exist. In our calculations $\bar{r}_7(b\bar{b}) = 1.6 \text{ fm}$ and $\bar{r}_5(c\bar{c}) = 1.8 \text{ fm}$ (in single-channel approximation) and their masses (without a hadronic shift) are $M(\Upsilon(7S)) \approx 11.24 \text{ GeV}$, $M(\psi(5S)) \approx 4.63 \text{ GeV}$. In Eqs. (3.6) and (3.8) their electronic widths, $\Gamma_{e^+e^-}(\Upsilon(7S)) = 0.11 \text{ keV}$, $\Gamma_{e^+e^-}(\psi(5S)) = 0.54 \text{ keV}$ are given (see Table III).

IV. CONCLUSIONS

The electronic widths of high-lying resonances in HQ are of special interest for the theory because they provide important information about the QCD confining potential at large distances.

Our calculations, performed with a relativistic Hamiltonian, show that three effects give rise to a decrease of the electronic widths of vector mesons:

(i) The asymptotic-freedom behavior of the vector coupling, which determines the GE potential, gives a decrease of the leptonic widths of about 70% for the $\Upsilon(nS)$ resonances

($n \leq 6$) and about 50% for the $\psi(nS)$ ($n \leq 4$) resonances.

(ii) The flattening of the confining potential at large distances gives an additional drop ($\sim 15\%$) in the leptonic widths of HQ but only for high excitations: $n \geq 4$ for the $\Upsilon(nS)$ family and $n \geq 3$ for the $\psi(nS)$ states. In this case good agreement with experiment is obtained for all $\Gamma_{e^+e^-}(\Upsilon(nS))$ ($n \leq 5$) and for $\Gamma_{e^+e^-}(J/\psi)$, $\Gamma_{e^+e^-}(\psi(3686))$.

(iii) If for some reason the GE interaction is totally switched off for resonances of large size, then the leptonic widths of very high excitations, like $\Upsilon(11019)$, $\psi(4040)$, and $\psi(4415)$, strongly decrease and appear to be in good agreement with the experimental data. These three resonances have large r.m.s. radii, $r_n(Q\bar{Q}) \gtrsim 1.2$ fm and their purely nonperturbative dynamics turns out to be rather simple. It is essential that here the string tension $\sigma(r)$ is taken just the same as in the light meson analysis of radial excitations [6].

(iv) The electronic widths and masses of the still unobserved resonances: $\Gamma_{e^+e^-}(\Upsilon(7S)) = 0.11$ keV ($M_7(b\bar{b}) \approx 11250$) and $\Gamma_{e^+e^-}(\psi(5S)) \approx 0.54$ keV ($M_5(c\bar{c}) \approx 4630$) are predicted.

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